Inflow versus outflow zero-temperature dynamics in one dimension

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It has been suggested that Glauber (inflow) and Sznajd (outflow) zero-temperature dynamics for the one-dimensional Ising ferromagnet with nearest-neighbor interactions are equivalent. Here we compare the two dynamics from the analytical and computational points of view. We use the method of mapping an Ising spin system onto the dimer RSA model and show that already this simple mapping allows us to see the differences between inflow and outflow zero-temperature dynamics. Then we investigate both dynamics with synchronous, partially synchronous, and random sequential updating using the Monte Carlo technique and compare both dynamics in the number of persistent spins, clusters, mean relaxation time, and relaxation time distribution.

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I. INTRODUCTION

The majority of natural phenomena observed in physics, biology, geology, social sciences, etc., are nonequilibrium processes. Unfortunately, the theory of nonequilibrium statistical mechanics is far less developed than its equilibrium counterpart. As a result, these ubiquitous phenomena are poorly understood [1]. The zero-temperature dynamics of simple models such as Ising ferromagnets provides interesting examples of nonequilibrium dynamical systems with many attractors (absorbing configurations, blocked configurations, zero-temperature metastable states) [2]. In this paper we focus on so-called single-spin-flip dynamics for the one-dimensional Ising ferromagnet. The best-known example of such dynamics for the Ising model is Glauber dynamics [3]. It can be viewed as "inflow" dynamics, since the center spin is influenced by its nearest neighbors [4]. Another type of dynamics, which can be called "outflow" dynamics, since the information flows from the center spin (or spins) to the neighborhood, has been introduced to describe opinion formation in social systems [5]. It has been suggested [6,7] that both dynamics for an Ising ferromagnet with nearest-neighbor interactions are equivalent, at least in one dimension. However, this seems to be true only in some particular cases. The aim of this paper is to compare generalized outflow and inflow dynamics for a chain of Ising spins and show in which cases these are equivalent and in which they differ.

It should be noticed here that the models studied in this paper are closely related to the majority-rule (MR) model introduced by Krapivsky and Redner [8]. In the MR model a selected group of G spins adapts to the state of the local majority and eventually the system reaches consensus (all spins up or all spins down). Interestingly, a system described by the MR model and an Ising spin system with outflow dynamics and random sequential updating (known as the Sznajd model) behave similarly in some aspects. For instance, the exit probability has almost the same, nontrivial dependence on the initial magnetization [4,8], in contrast to the linear voter model [8,9,11].

However, both inflow and outflow dynamics with random sequential updating and the MR model belong to a very general class of voter models (VMs). Following Liggett [9,10], VM models are continuous-time Markov processes, which are described by specifying the rates at which the system changes from one configuration to another. Changes are generally local, in that only several sites change state at any given time, and the rates for such transitions depend on the configurations near those sites. The inflow dynamics has already been reformulated in terms of a linear voter model, which is exactly soluble [9,13]. Probably the outflow dynamics could be reformulated in terms of a nonlinear voter model. Unfortunately, except for the linear voter model case and some very special cases of nonlinear voter models, the exact symmetry of voter models places them beyond the reach of currently available techniques for rigorous mathematical analysis, but at least some Monte Carlo simulation results are known [12]. Thus, it would be interesting to reformulate the outflow dynamics in terms of a nonlinear voter model and check if this is one of the few fortunate solvable cases or at least if there are some Monte Carlo results for related voter models. Although this is an interesting and important task we leave it for future work and concentrate here rather on comparisons of both dynamics under various updating schemes.

In Secs. II and III we recall ideas of inflow and outflow dynamics and formulate the generalized versions of both dynamics. We take both dynamics under a common roof, reformulating them without using the concept of energy. In Sec. IV we use the illuminating method of mapping the Ising spin system onto the dimer RSA model and make simple mean-field-like calculations to show the difference between the dynamics. In Sec. V we present Monte Carlo results for both dynamics with several kinds of updating including synchronous, partially synchronous, and random sequential updating. The summary and conclusions are the subjects of Sec. VI of the paper.

II. INFLOW DYNAMICS

The best-known example of such dynamics for the Ising model is Glauber dynamics. Within Glauber dynamics, in a broad sense, each spin is flipped $S_i(\tau) \rightarrow -S_i(\tau+1)$ with a rate

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$W(\delta E)$ per unit time and this rate is assumed to depend only on the energy difference implied in the flip [2]. The two most usual choices of flipping rates in the case of discrete updates are the heat-bath and Metropolis algorithms; both obey the detailed balance condition

$$\frac{W(\delta E)}{W(-\delta E)} = \exp(-\beta\delta E). \tag{1}$$

Recently it was shown [2] that there is a vast family of dynamical rates, besides these two choices, which obeys the condition (1). Among them the class of zero-temperature dynamics defined as

$$W(\delta E) = \begin{cases} 1 & \text{if } \delta E < 0, \\ W_0 & \text{if } \delta E = 0, \\ 0 & \text{if } \delta E > 0, \end{cases} \tag{2}$$

is very interesting. The zero-temperature limits of the heat-bath and Metropolis rates are, respectively, $W_0^{HB}=1/2$ and $W_0^M=1$. For any nonzero value of the rate W_0 corresponding to free spins, the dynamics belongs to the universality class of the zero-temperature Glauber model. This is a prototypical example of phase ordering by domain growth (coarsening). The typical size of ordered domains of consecutive ↑ and ↓ spins grows as $L(t) \sim t^{1/2}$. The particular value $W_0=0$ corresponds to the constrained zero-temperature Glauber dynamics ([2] and references therein). In the constrained zero-temperature Glauber dynamics, the only possible moves are flips of isolated spins and the system therefore eventually reaches a blocked configuration, where there is no isolated spin [2]. Very interesting results for the zero-temperature Glauber dynamics have also been obtained using computer simulations [14–17].

In out-of-equilibrium systems, there is usually no energy function and the system is only defined by its dynamical rules [18]. This is also the case of the sociophysics Sznajd model. For this reason we reformulate the definition of the zero-temperature Glauber dynamics for the Ising ferromagnet without using the concept of energy in the following way:

$$S_i(\tau+1) = \begin{cases} 1 & \text{if } \sum_{NN} S_{NN} > 0, \\ -S_i(\tau) \text{ with probability } W_0 & \text{if } \sum_{NN} S_{NN} = 0, \\ -1 & \text{if } \sum_{NN} S_{NN} < 0, \end{cases} \tag{3}$$

where $\Sigma_{NN} S_{NN}$ denotes the sum over nearest neighbors.

In one dimension, which is the case of this paper, the above definition can be obviously written as

$$S_i(\tau+1) = \begin{cases} 1 & \text{if } S(\tau)_{i-1} + S(\tau)_{i+1} > 0, \\ -S_i(\tau) \text{ with probability } W_0 & \text{if } S(\tau)_{i-1} + S(\tau)_{i+1} = 0, \\ -1 & \text{if } S(\tau)_{i-1} + S(\tau)_{i+1} < 0. \end{cases} \tag{4}$$

III. OUTFLOW DYNAMICS

Outflow dynamics was introduced to describe opinion change in a society. The idea is based on the fundamental social phenomenon called "social validation." However, in this paper we do not focus on social applications of the model (for interested readers, reviews can be found in [19–22]). On the contrary, here we investigate the dynamics from the theoretical point of view.

In the original model a pair of neighboring spins S_i and S_{i+1} was chosen and if $S_iS_{i+1}=1$ the two neighbors of the pair followed its direction, i.e., $S_{i-1} \rightarrow S_i$ $(=S_{i+1})$ and $S_{i+2} \rightarrow S_i$ $(=S_{i+1})$. Such a rule was also used in all later papers dealing with the one-dimensional case of the model. However, the case in which $S_iS_{i+1}=-1$ was noted as far less obvious. For example, in the original paper in the case of $S_iS_{i+1}=-1$, $S_{i-1} \rightarrow S_{i+1}$ and $S_{i+2} \rightarrow S_i$. However, it was noticed in several papers that such a rule is unrealistic in a model trying to represent the behavior of a community. Moreover, it can be seen from the following two rules that the original Sznajd model with both ferromagnetic and antiferomagnetic rules is equivalent to the single simple rule that every spin takes the direction of its next-nearest neighbor independently of the product S_iS_{i+1}.

Ferromagnetic rule. If $S_i(\tau)S_{i+1}(\tau)=1$ then $S_{i-1}(\tau+1) \rightarrow S_i(\tau)$ and $S_{i+2}(\tau+1) \rightarrow S_{i+1}(\tau)$ is equivalent to the rule: that if $S_i(\tau)S_{i+1}(\tau)=1$ then $S_{i-1}(\tau+1) \rightarrow S_{i+1}(\tau)$ if $S_{i+2}(\tau+1) \rightarrow S_i(\tau)$.

Antiferromagnetic rule. If $S_i(\tau)S_{i+1}(\tau)=-1$ then $S_{i-1}(\tau+1) \rightarrow S_{i+1}(\tau)$ and $S_{i+2}(\tau+1) \rightarrow S_i(\tau)$.

Thus, the two rules above can be rewritten as a simple single rule: $S_{i-1}(\tau+1) \rightarrow S_{i+1}(\tau)$ and $S_{i+2}(\tau+1) \rightarrow S_i(\tau)$.

In later papers we proposed two modifications of the model in which the antiferromagnetic rule was replaced by one of rules described below.

Modification 1. If $S_i(\tau)S_{i+1}(\tau)=-1$, then $S_{i-1}(\tau+1) \rightarrow S_{i-1}(\tau)$ and $S_{i+2}(\tau+1) \rightarrow S_{i+2}(\tau)$.

Modification 2. If $S_i(\tau)S_{i+1}(\tau)=-1$, then $S_{i-1}(\tau+1) \rightarrow -S_{i-1}(\tau)$ and $S_{i+2}(\tau+1) \rightarrow -S_{i+2}(\tau)$ with probability 1/2.

The generalized dynamics including the two modifications above, can be written as

$$S_i(\tau+1) = \begin{cases} 1 & \text{if } S_{i+1}(\tau) + S_{i+2}(\tau) > 0, \\ -S_i(\tau) \text{ with probability } W_0 & \text{if } S_{i+1}(\tau) + S_{i+2}(\tau) = 0, \\ -1 & \text{if } S_{i+1}(\tau) + S_{i+2}(\tau) < 0. \end{cases} \tag{5}$$

It is easy to see that modification 1 corresponds to $W_0^1 = 0$ and modification 2 to $W_0^2 = 1/2$.

IV. MAPPING ONTO THE DIMER MODEL

As was mentioned in previous sections, for $W_0 = 0$ the system under inflow (Glauber) dynamics described by the formula (4) eventually reaches a blocked configuration, where there is no isolated spin. On the other hand the system under outflow dynamics described by (5) always reaches a ferromagnetic steady state. Thus, for $W_0 = 0$ the difference between outflow and inflow dynamics is obvious. Nevertheless, within the mean-field approach [23] and Galam's unifying frame [6] both dynamics are equivalent, i.e., there is no difference between outflow and inflow dynamics, even for $W_0 = 0$.

Here we use the illuminating method of mapping the Ising spin system onto the dimer RSA model, which has already been done for the inflow dynamics [2]:

$$X_i = S_i S_{i+1} = 1 \Rightarrow \circ,$$

$$X_i = S_i S_{i+1} = -1 \Rightarrow \bullet. \tag{6}$$

In the case of inflow dynamics the following transitions, which change the state of the system, are possible:

spins	particles
$\downarrow\uparrow\downarrow \rightarrow \downarrow\downarrow\downarrow$	$\bullet\bullet \rightarrow \circ\circ$
$\uparrow\downarrow\uparrow \rightarrow \uparrow\uparrow\uparrow$	$\bullet\bullet \rightarrow \circ\circ$
$\downarrow\uparrow\uparrow \xrightarrow{W_0} \downarrow\downarrow\uparrow$	$\bullet\circ \xrightarrow{W_0} \circ\bullet$
$\uparrow\downarrow\downarrow \xrightarrow{W_0} \uparrow\uparrow\downarrow$	$\bullet\circ \xrightarrow{W_0} \circ\bullet$
$\downarrow\downarrow\uparrow \xrightarrow{W_0} \downarrow\uparrow\uparrow$	$\circ\bullet \xrightarrow{W_0} \bullet\circ$
$\uparrow\uparrow\downarrow \xrightarrow{W_0} \uparrow\downarrow\downarrow$	$\circ\bullet \xrightarrow{W_0} \bullet\circ$

Thus, after mapping there are only two types of transitions for inflow dynamics: $\bullet\bullet \rightarrow \circ\circ$ and $\circ\bullet \overset{W_0}{\leftrightarrow} \bullet\circ$. This mapping shows at once that for $W_0 = 0$ the dynamics is fully irreversible, in the sense that each spin flips at most once during the whole history of the sample.

It should be noticed that if we map the system under outflow dynamics onto the dimer model we have to take into account four particles, because changing the border spin influences the next particle. In this case four types of transitions are possible: $\circ\bullet\circ \rightarrow \circ\circ\bullet$, $\circ\bullet\bullet \rightarrow \circ\circ\circ$, $\bullet\circ\bullet \overset{W_0}{\leftrightarrow} \bullet\bullet\circ$, and $\bullet\bullet\bullet \overset{W_0}{\leftrightarrow} \bullet\circ\circ$ (to make it more clear the flipped spins are denoted by double arrows in the table below):

spins	particles
$\downarrow\downarrow\Uparrow\uparrow \rightarrow \downarrow\downarrow\Downarrow\uparrow$	$\circ\bullet\circ \rightarrow \circ\circ\bullet$
$\uparrow\uparrow\Downarrow\downarrow \rightarrow \uparrow\uparrow\Uparrow\downarrow$	$\circ\bullet\circ \rightarrow \circ\circ\bullet$
$\downarrow\downarrow\Uparrow\downarrow \rightarrow \downarrow\downarrow\Downarrow\downarrow$	$\circ\bullet\bullet \rightarrow \circ\circ\circ$
$\uparrow\uparrow\Downarrow\uparrow \rightarrow \uparrow\uparrow\Uparrow\uparrow$	$\circ\bullet\bullet \rightarrow \circ\circ\circ$
$\downarrow\uparrow\Uparrow\downarrow \xrightarrow{W_0} \downarrow\uparrow\Downarrow\downarrow$	$\bullet\circ\bullet \xrightarrow{W_0} \bullet\bullet\circ$
$\uparrow\downarrow\Downarrow\uparrow \xrightarrow{W_0} \uparrow\downarrow\Uparrow\uparrow$	$\bullet\circ\bullet \xrightarrow{W_0} \bullet\bullet\circ$
$\uparrow\downarrow\Uparrow\downarrow \xrightarrow{W_0} \uparrow\downarrow\Downarrow\downarrow$	$\bullet\bullet\bullet \xrightarrow{W_0} \bullet\circ\circ$
$\downarrow\uparrow\Downarrow\uparrow \xrightarrow{W_0} \downarrow\uparrow\Uparrow\uparrow$	$\bullet\bullet\bullet \xrightarrow{W_0} \bullet\circ\circ$
$\downarrow\uparrow\Downarrow\downarrow \xrightarrow{W_0} \downarrow\uparrow\Uparrow\downarrow$	$\bullet\bullet\circ \xrightarrow{W_0} \bullet\circ\bullet$
$\uparrow\downarrow\Uparrow\uparrow \xrightarrow{W_0} \uparrow\downarrow\Downarrow\uparrow$	$\bullet\bullet\circ \xrightarrow{W_0} \bullet\circ\bullet$
$\uparrow\downarrow\Downarrow\downarrow \xrightarrow{W_0} \uparrow\downarrow\Uparrow\downarrow$	$\bullet\circ\circ \xrightarrow{W_0} \bullet\bullet\bullet$
$\downarrow\uparrow\Uparrow\uparrow \xrightarrow{W_0} \downarrow\uparrow\Downarrow\uparrow$	$\bullet\circ\circ \xrightarrow{W_0} \bullet\bullet\bullet$

This mapping shows that for $W_0 = 0$ the outflow dynamics consists of two processes—diffusion of • particles in the sea of ∘ and annihilation of •• pairs. Thus our model for W_0 with random sequential updating reduces to the analytically solvable reaction-diffusion system $A + A \rightarrow 0$ (denoting the empty place by ∘ and the A particle by •).

For $W_0 \geq 0$ we can also easily use the mean-field approach (MFA). Mean-field results for the outflow dynamics without dimer mapping can be found in [23]. Within dimer mapping we take into account correlations between pairs of nearest neighbors. Thus if we apply the MFA to the mapped system we expect more correct results than are obtained within the MFA without mapping.

Let us denote the number of • particles by N_b and $\frac{N_b}{N} = b$.

In our case, in one time step τ, only two events are possible—the number of • particles decreases by $2/N$ with probability $\gamma(b)$ or remains constant.

For the inflow dynamics

$$\gamma^{in}(b) = b^2 \tag{7}$$

and for the outflow dynamics

$$\gamma^{out}(b) = (1-b)b^2 + W_0 b^3 = b^2[1 - b(1 - W_0)]. \tag{8}$$

It is seen that the above results are not precise, since there is no dependence between $\gamma^{in}(b)$ and W_0 for the inflow dynamics and the only stable steady state in this case is $b=0$, i.e., the ferromagnetic state, which is true as long as $W_0>0$. However, as has been noticed this result is not correct for $W_0=0$. The same results can be obtained using the mean-field approach without mapping.

However, for the outflow dynamics the MFA with dimer mapping gives better results than the basic MFA presented in [23]. This is understandable, because in this case pairs of neighboring spins cause the changes in the system.

For $W_0=0$ there are two steady states, $b=0$, i.e., the ferromagnetic state, and $b=1$, i.e., the antiferromagnetic state. For $b\neq 0$ and $b\neq 1$ $\gamma^{out}(b)>0$ which implies that $b=0$ is an unstable steady state, while $b=1$ is a stable steady state. This result is in agreement with computer simulations [5]. For $W_0=1$ there is only one ferromagnetic steady state, which is also confirmed by the computer simulations [4].

As we see the differences between outflow and inflow dynamics are already seen if we apply the mean-field approach with mapping of the pairs of spins into single particles. In the next section we present simulation results which show more differences between these two dynamics.

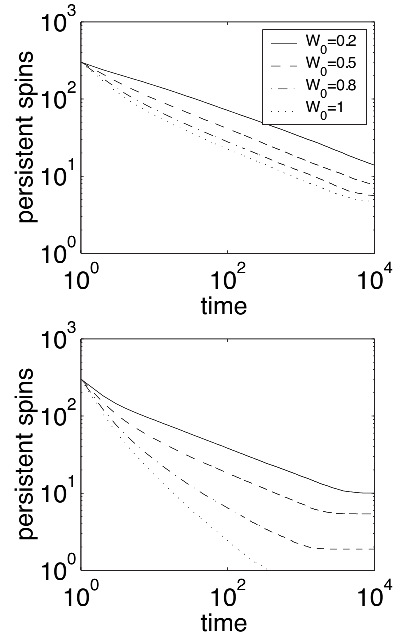

FIG. 1. The change in time of the number of persistent spins on the chain $N=300$ for random sequential updating (i.e., $c=0$) for the inflow dynamics (upper panel) and outflow dynamics (lower panel).

V. SIMULATION RESULTS

The spin updating within both dynamics can be sequential or parallel. Within the parallel (or in other words synchronous) updating the system state at time step $t+1$ is given by its state at time step t. At every time step t we go systemically through the whole lattice and change spins according to the appropriate rule. In the random sequential (or in other words asynchronous) updating in each time step only one spin is selected at random and it adapts to its neighborhood. One Monte Carlo step (MCS) in this case consists of N time steps, while in the case of parallel updating one MCS is equivalent to a single time step.

In this paper we compare both dynamics for random sequential updating, parallel updating, and partially parallel updating. From now on we call the last case c-parallel updating. Within this updating the randomly chosen fraction c of spins is updated synchronously. Of course $c=1$ corresponds to parallel updating and $c=0$ to random sequential updating.

A. The number of persistent spins

One of the main quantities of interest in the nonequilibrium dynamics of spin systems at zero temperature is the fraction of spins $P(t)$ that persist in the same state up to some later time $t=N\tau$ (i.e., measured in Monte Carlo steps) [24,25]. In this paper we measure the fraction of persistent spins for both outflow and inflow dynamics with c-parallel updating for different values of c. The initial configuration consists of a randomly distributed fraction $p_+(0)$ of up spins. The number of persistent spins for the outflow dynamics with $W_0=0$ and random sequential updating has already been investigated by Stauffer and Oliveira [26] and found to agree with results for the inflow dynamics, i.e., decays with time t as $1/t^{-3/8}$. However, it was found that in higher dimensions the exponents for inflow and outflow dynamics are different [26]. Here we investigate the case of the Ising spin chain more carefully, i.e., for different values of W_0 and c.

The first difference between inflow and outflow dynamics is already seen for random sequential updating, i.e., $c=0$. For both dynamics the number of persistent spins decays initially as a power law $\sim t^{-\theta}$. However, for inflow dynamics the exponent is independent of W_0 until $W_0>0$, while for outflow dynamics the exponent is W_0 dependent, $\theta=\theta(c)$ (see Fig. 1). Moreover, for inflow dynamics the power law describes properly the decay of the number of persistent spins for the whole range of time, while within the outflow dynamics it is valid only for t smaller than a certain value of time $t^*(W_0)$ dependent on the flipping probability W_0. For $W_0\to 0$ we obtain $t^*(W_0)\to\infty$ and the evolution of the number of persistent spins is the same for outflow and inflow dynamics in agreement with the results obtained by Stauffer and Oliveira [26].

More differences are seen for partially synchronous updating with $c>0$. At each elementary time step τ the fraction c of spins is chosen randomly and the chosen group is changed synchronously. In such a case we have noticed that the number of persistent spins still decays as a power law for the inflow dynamics. However, for the outflow dynamics the power law is no longer valid. The number of persistent spins decays very fast in this case (see Fig. 2).

We may conclude this subsection in the following – the number of persistent spins is c sensible only for the outflow dynamics. For $W_0>0$ and any value of c the number of persistent spins in the inflow dynamics is described by the power law with nearly the same exponent.

B. The number of clusters

Probably the most natural way to investigate the relaxation process of the consensus dynamics is to look at the

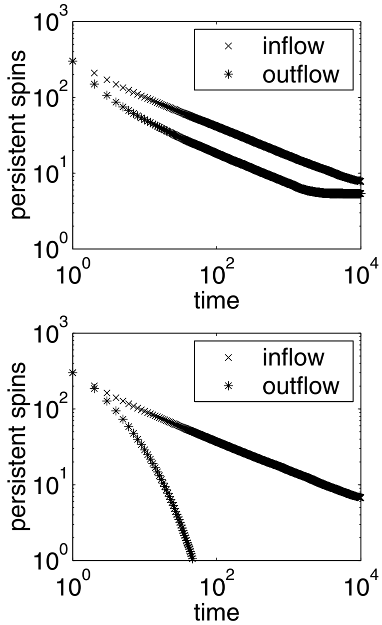

FIG. 2. The change in time of the number of persistent spins on the chain $N=300$ for partially synchronous updating for $W_0=1/2$. Upper panel presents results for $c=0$ and lower for $c=0.2$. It is seen that for $c>0$ (bottom case) the number of persistent spins decays very fast and cannot be described by a power law.

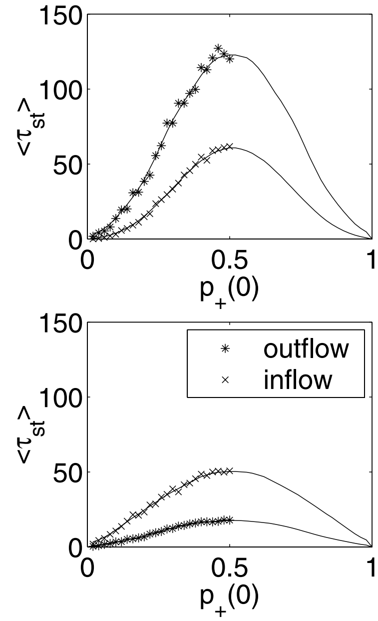

FIG. 3. The mean relaxation times from a random initial state consisting of $p_+(0)$ randomly distributed up spins for $W_0=0.2$. Upper panel corresponds to synchronous updating $c=1$, and bottom panel to $c=0.2$. It is seen that the relaxation under outflow dynamics is slower then under inflow for synchronous updating. On the contrary, the relaxation is much slower under the inflow dynamics than under the outflow dynamics for small c.

number of clusters (or number of domain walls between neighboring opposite spins) over time. A cluster consists of a group of spins, each of which is a nearest neighbor to at least one other spin in the cluster, with all spins having the same orientation. With such a definition consensus is reached when only one cluster is present in the system or when there is no domain wall. Because of the similarity of both dynamics (inflow and outflow) with random sequential updating to the VM, we expected, and have verified numerically, that the density of domain walls (as well as the number of clusters) decays as $t^{-1/2}$, analogously with the results for the MR model [8].

Moreover, for both inflow and outflow dynamics with c-parallel updating the number of clusters (and the number of domain walls) monotonically decays as $t^{-1/2}$ for any value of c. This result shows that the variation of the number of clusters in time, although is a very intuitive and natural measure of the relaxation, is not a good quantity for dynamics comparison.

C. The mean relaxation time

The differences between the dynamics can be observed clearly if we look at the mean relaxation time as a function of the initial fraction of randomly distributed up spins $p_+(0)$. Within 0-parallel updating (i.e., random sequential updating) the relaxation is much slower for the inflow dynamics than for the outflow dynamics. This is also true for the c-parallel updating with small c. On the contrary, within 1-parallel updating (i.e., synchronous updating) the relaxation under outflow dynamics is slower than under inflow (see Fig. 3).

In general, the relaxation time decays with W_0 growth, but the dependence between the mean relaxation time and W_0 is different for outflow and inflow dynamics. Two examples for $c=0.2$ and 0.5 for several values of W_0 are shown in Figs. 4 and 5, respectively. It can be noted that, for example, for $c=0.5$ and $W_0=0.8$ the dependence between the mean relaxation time and the initial concentration of up spins $p_+(0)$ is nearly the same.

In Fig. 6 we have presented the dependence between the mean relaxation times from a random initial state consisting of 50% randomly distributed up spins (maximal waiting time) and the flipping probability W_0 for the inflow and outflow dynamics. It is seen that the dependence on c is much stronger for the outflow dynamics. For the inflow dynamics the mean relaxation time is almost the same for all values of c. On the other hand for a given value of c the dependence between $\langle\tau\rangle$ and W_0 is stronger for the inflow dynamics.

D. The distribution of waiting times

In the paper [23] the mean-field approach for the outflow dynamics with $W_0=0$ was presented and the distribution of waiting times needed to reach the stationary state was found. Recall that for δ initial conditions the distribution of waiting times has an exponential tail [23]:

$$P_st^{>}(\tau) \approx \frac{6}{4}(1-m_0^2)e^{-2\tau}, \quad \tau \to \infty . \tag{9}$$

Monte Carlo simulations confirm this prediction both on the complete graph and on the chain. In this paper we have

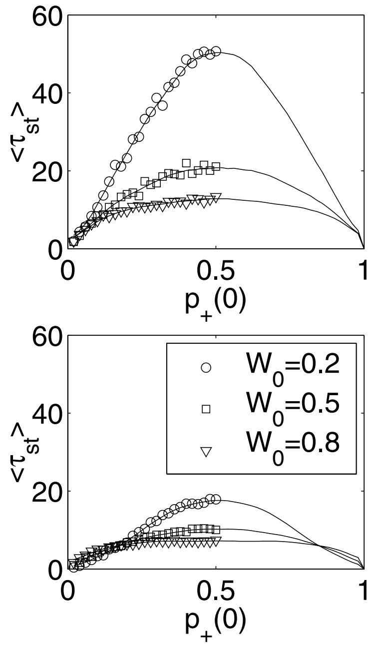

FIG. 4. The mean relaxation times from the random initial state consisting of $p_{+}(0)$ randomly distributed up spins for $c=0.2$ for the inflow (upper panel) and outflow (bottom panel) dynamics.

checked also the distribution of waiting times for different values of W_0 and c for both outflow and inflow dynamics. The distribution of waiting times has an exponential tail for any value of W_0 and c, although the exponent depends on these parameters. The example for $c=0$, showing comparison between inflow and outflow dynamics, is shown in Fig. 7.

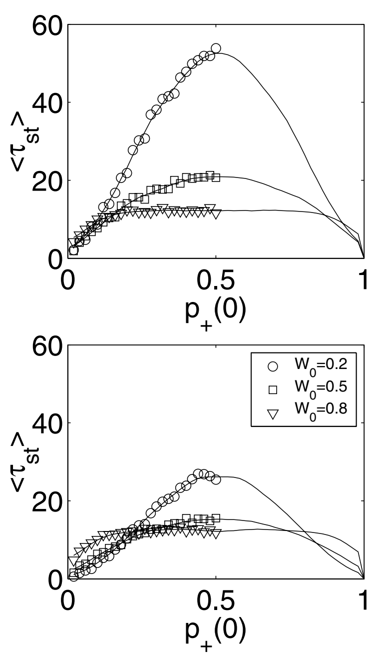

FIG. 5. The mean relaxation times from the random initial state consisting of $p_{+}(0)$ randomly distributed up spins for $c=0.5$ for the inflow (upper panel) and outflow (bottom panel) dynamics.

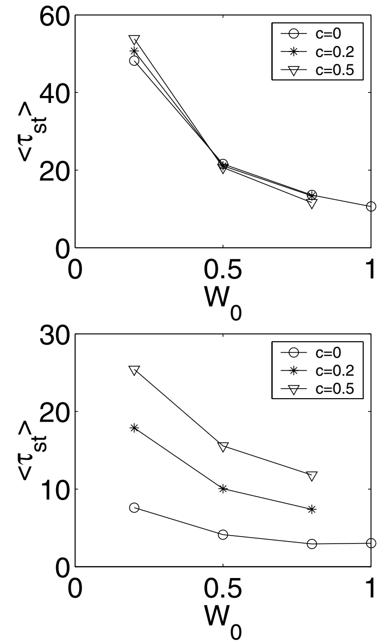

FIG. 6. The dependence between the mean relaxation times from the random initial state consisting of 50% randomly distributed up spins and the flipping probability W_0 for the inflow (upper panel) and outflow (bottom panel) dynamics for different values of c. It is seen that the dependence on c is much stronger for the outflow dynamics. On the other hand for a given value of c, the dependence between $\langle \tau \rangle$ and W_0 is stronger for the inflow dynamics.

VI. CONCLUSIONS

It has been suggested [6,7] that zero-temperature outflow and inflow dynamics for an Ising ferromagnet with nearest-

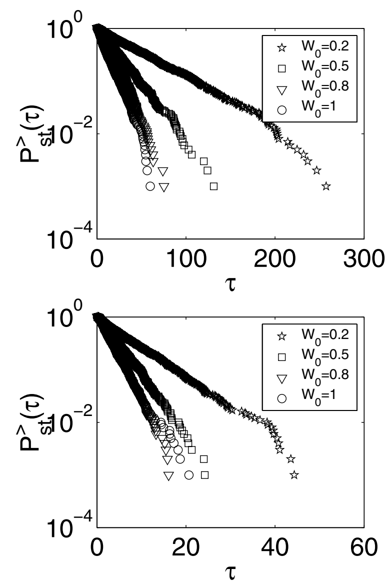

FIG. 7. Probabilities of reaching a steady state in time larger than τ on the chain $L=200$. The distribution of waiting times has an exponential tail for both dynamics—inflow (upper panel) and outflow (lower panel).

neighbor interactions are equivalent in one dimension. However, it is certainly not true for $W_0=0$. This particular value corresponds to the constrained zero-temperature Glauber dynamics where the only possible moves are flips of isolated spins and the system therefore eventually reaches a blocked configuration, where there is no isolated spin [2]. This can be also easily shown using the method of mapping the Ising spin system onto the RSA dimer model. On the other hand, the outflow dynamics leads to a ferromagnetic steady state for any value of W_0. This observation motivated us to compare both dynamics more carefully. We have made Monte Carlo simulations for both dynamics using random sequential updating, parallel updating, and c-parallel updating (a randomly chosen fraction c of spins is updated synchronously). We have measured, for different values of W_0 and c, the distribution of waiting times, the mean waiting time, the decay of the number of clusters, and the number of persistent spins in time.

A qualitative difference between inflow and outflow dynamics is not visible either in the number of clusters in time or in the distribution of waiting times. However, it should be noticed that the relaxation time is different for the two both dynamics. Nevertheless, for both dynamics the distribution of waiting times has an exponential tail and the number of clusters decays as $t^{-1/2}$ for any value of $W_0>0$ and c.

Differences between the dynamics appear if we look at the dependence between the mean relaxation time and the initial concentration of randomly distributed up spins for different values of W_0 and c. For $c=0$, which corresponds to random sequential updating, the mean relaxation time is shorter for the outflow dynamics (e.g., for $W_0=0.2$ and $p_0=0.5$ it is about ten times shorter) than for inflow. The mean relaxation time $\langle\tau\rangle$ decreases with W_0 growth for both dynamics, but the dependence between $\langle\tau\rangle$ and W_0 is different for outflow and inflow dynamics. Generally the mean relaxation time decays faster with growing W_0 for the inflow dynamics for any value of c. Moreover, with growing c the dependence between the mean relaxation time for the inflow dynamics and the outflow dynamics vanishes. As a result, for some values of c and W_0 (e.g., $c=0.5$ and $W_0=0.8$) the dependence between the mean relaxation times and the initial concentration of up spins is identical. Of course this suggests that for some values of parameters W_0 and c the relaxation under outflow dynamics is faster than under inflow dynamics. This is indeed true. In the case of $c=1$ (parallel updating), the relaxation is faster under the inflow dynamics for any value of W_0.

The second quantity that behaves differently for the two dynamics is the number of persistent spins in time. Mainly differences are seen for partially synchronous updating with $c>0$. In such a case we have noticed that the number of persistent spins decays as a power law for the inflow dynamics (as in the case of $c=0$). However, for the outflow dynamics the power law is no longer valid. The number of persistent spins decays very fast in this case.

Concluding, the inflow and outflow dynamics differ very clearly even in one dimension. There is an obvious, very strong difference for $W_0=0$, but also for $W_0>0$ the two dynamics are qualitatively different. In the case of random sequential updating the relaxation under outflow dynamics is much faster than under inflow dynamics. On the contrary in the case of parallel updating the outflow dynamics is much slower than the inflow.

In closing this paper we should mention that the outflow dynamics with $W_0=0$ with synchronous updating was investigated earlier and it was found that in such a case the possibility of reaching a consensus is reduced quite dramatically [27]. Also the number of persistent spins varies with c only for the outflow dynamics. For $W_0>0$ and any value of c the number of persistent spins in the inflow dynamics is described by a power law with nearly the same exponent.

Generally the outflow dynamics is much more influenced by the type of updating than the inflow dynamics. We believe that this result especially is very important in the various interdisciplinary applications of the zero-temperature single-spin-flip dynamics.

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